PROBLEMS
CLASS 5
Chapter 3

Question Q3.12

The centripetal acceleration is $a = \frac{v^2}{R}$. When the speed is doubled, the acceleration increases by a factor of 4. When the radius is halved, the acceleration doubles.

Exercise 3.29

(a) Note first that 1 hour = 3600 sec. Therefore the radial (centripetal) acceleration of an object on the earth's equator is

$$R := 6380 \cdot km \qquad R = 6.38 \times 10^6\,m \qquad T := 24 \cdot 3600 \cdot sec \qquad T = 8.64 \times 10^4\,s$$

$$v := \frac{2 \cdot \pi \cdot R}{T} \qquad v = 463.967\frac{m}{s}$$

$$a := \frac{v^2}{R} \qquad a = 0.034\frac{m}{s^2}$$

(b) If a = g, then $g = \frac{v^2}{R} \quad \Rightarrow \quad v = \sqrt{Rg}$. The period T is $T = \frac{2\pi R}{v}$; hence

$$v := \sqrt{g \cdot R} \qquad v = 7.91 \times 10^3\,\frac{m}{s}$$

$$T := \frac{2 \cdot \pi \cdot R}{v} \qquad T = 5.07 \times 10^3\,s$$

Exercise 3.33

The magnitude of the centripetal acceleration is always

$$v := 7 \cdot \frac{m}{sec} \qquad R := 14.0 \cdot m$$

$$a := \frac{v^2}{R} \qquad a = 3.5\frac{m}{s^2}$$

(a) At the lowest point, the direction of the centripetal acceleration is up (towards the center of the circle).

(b) At the top, the direction is down.

(c) The period (time for one revolution is)

$$T := \frac{2 \cdot \pi \cdot R}{v} \qquad T = 12.6s$$

Problem 3.50

(a) Relative to the ground, we need consider only the horizontal speed of the bird. Since the bird is moving horizontally in uniform circular motion, that speed is

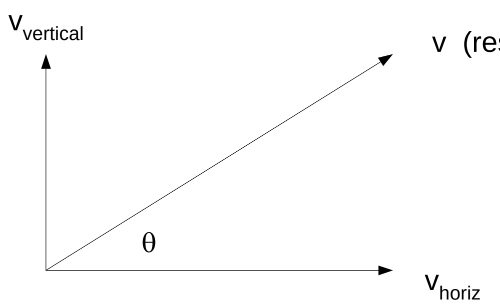
$$R := 8.0 \cdot m \qquad T := 5.0 \cdot sec$$

$$v := \frac{2 \cdot \pi \cdot R}{T} \qquad v = 10.05 \frac{m}{s}$$

(b) The bird is moving up at a constant speed, so the only acceleration is the centripetal acceleration, directed towards the center of the circle. The magnitude is

$$a := \frac{v^2}{R} \qquad a = 12.6 \frac{m}{s^2}$$

(c) The horizontal speed is just the speed in uniform circular motion, found in part (a).



Evidently the angle the resultant velocity makes with the horizontal is given by

$$v_{horiz} := 10.05 \cdot \frac{m}{sec} \qquad v_{vert} := 3 \cdot \frac{m}{sec}$$

$$\theta := \operatorname{atan}\left(\frac{v_{vert}}{v_{horiz}}\right) \qquad \theta = 16.6 deg$$

Problem 3.79

(a) We assume that the number of revolutions per minute (rpm) is constant. If so, then as we move from the outer edge of centrifuge to the half-way point, **both** the velocity and the radius change. If *T* is the period for 1 revolution, then

$$v = \frac{2\pi R}{T} \qquad \text{and} \qquad a_c = \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2}.$$

The book derives this result. Be sure you can too—I will sometimes insist that you start from the first form. If the rpm value stays the same, so does the period *T*; and since the new radius is half the original, we have

$$a_c = 2.5g \text{ at the half-way point.}$$

(b) The easy way to do this part is to notice that if *n* is the number of revolutions per **second**, then

$$n = \frac{1}{T};$$

that is, if the object takes two seconds to make one revolution (T = 2 sec), then it makes half a revolution in 1 sec (n = 0.5 rev. per sec). Hence $a_c = 4\pi^2 R\, n^2$. We write this equation for the earth and for mercury (where the gravitational acceleration is 0.378 g):

$$4\pi^2 R n_{\text{earth}}^2 = 5g$$
$$4\pi^2 R n_{\text{Mercury}}^2 = 5\left(0.378\ \text{g}\right)$$

where as before, R is the radius of the centrifuge. If we now divide the bottom equation by the top one, we have

$$\frac{4\pi^2 R\, n_{\text{Mercury}}^2}{4\pi^2 R\, n_{\text{earth}}^2} = 0.378$$
$$\frac{n_{\text{Mercury}}^2}{n_{\text{earth}}^2} = 0.378$$
$$n_{\text{Mercury}} = \sqrt{0.378}\ n_{\text{earth}} = 0.615\, n_{earth}$$

Note that since we are taking a ratio, it doesn't matter whether n has units of rev/sec or rev/min (rpm).