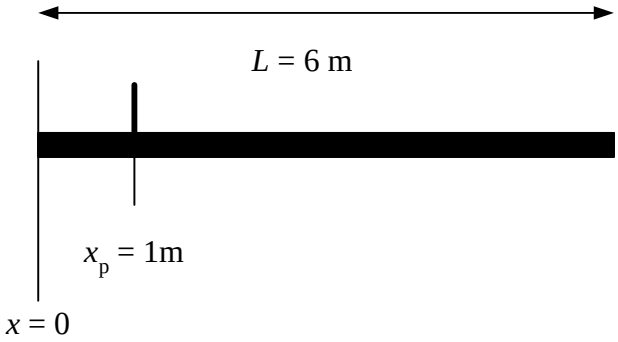
**Center of mass example**

A 100 kg canoeist is in a 30 kg canoe. The canoe is $L = 6$ m long. Initially the person is 1m from the left end of the canoe, as shown in the drawing. S/he moves to 1 m from the right end of the canoe (or 5 m from the left end). Assume there is no friction between the canoe and the water, so that there are no external horizontal forces on the canoe-person system. Find how far the canoe moves.

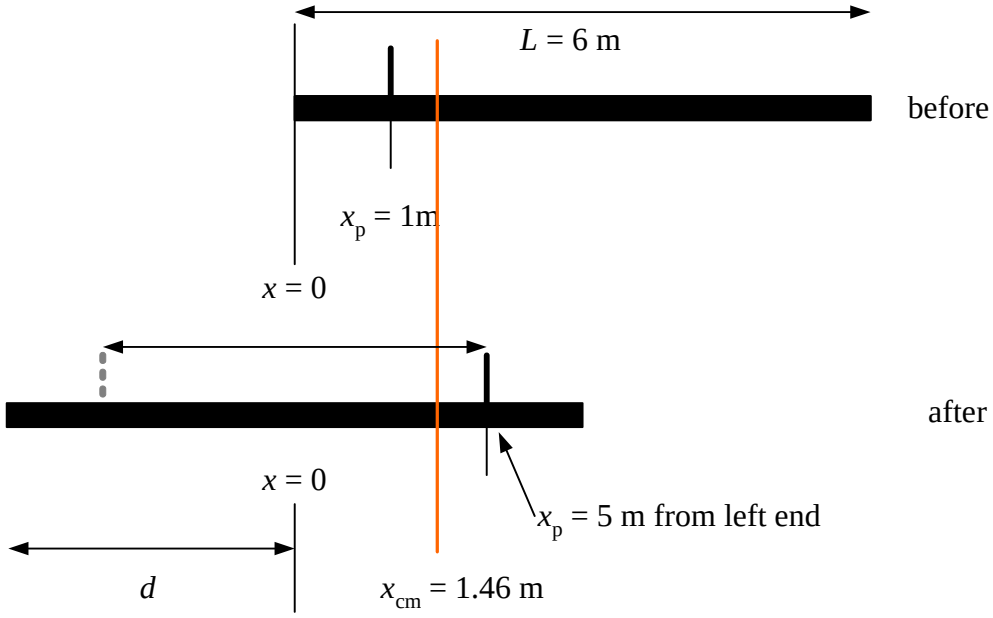
We choose a coordinate system in which the origin, $x = 0$, is at the left end of the canoe. Initially, the situation looks like so:




We assume that we can treat the canoe as if its mass were concentrated at a point at its center. So initially, the positions of the canoeist and the center of the canoe are $x_p = 1$ m and $x_c = L/2 = 3$ m. With that assumption, we can find the center of mass:

$$x_{cm} = \frac{m_p x_p + m_c x_c}{m_p + m_c} = \frac{100 \text{ kg (1 m)} + 30 \text{ kg (3 m)}}{130 \text{ kg}} = 1.46 \text{ m}$$

Now, suppose the person walks 4 m along the canoe, and so is 1 m from the right end or 5 m from the left end. At the same time, the canoe will move some (unknown) distance *d* in the other direction. A before-after picture will be




Since there are no horizontal external forces, the center of mass does not move—it stays fixed at 1.46 m. The canoeist moves to the right; and the canoe moves an unknown distance *d* to the left. The drawing is approximately to scale; note that the distance from the canoeist to the center of mass is the same.

To find *d*, we must first find the new positions of the canoeist, and of the center of the canoe, with respect to $x = 0$. Since the person moves 4 m to the right (and so is 5 m from the left end of the canoe), while the canoe moves a distance *d*, the new position of the canoeist is $x'_p = 5 + d$. Similarly, the center of the canoe is now at a position $x'_c = 3 + d$. We expect, of course, that *d* will be a negative number (to the left). So the equation for the center of mass after the canoeist has moved is

$$x_{cm} = 1.46 \text{ m} = \frac{m_p x'_p + m_c x'_c}{m_p + m_c} = \frac{m_p(5+d) + m_c(3+d)}{m_p + m_c}$$

I will leave the arithmetic it to you. You should be able to show that $d = -3.08$ m.