

# Selected Answers

## CHAPTER 1

### Section 1.1

1. 0.6%,  $\pi \approx 3.1604938$ . 3.  $1.4146296 \approx \sqrt{2}$ ,  $42.42638 \approx 30\sqrt{2}$ . 5a)  $72^\circ$ ,  $54^\circ$ ,  $108^\circ$ ,  $36^\circ$ ,  $108^\circ$ ,  $72^\circ$ ,  $36^\circ$ . 5c)  $1+x = (1+\sqrt{5})/2$ . 6a)  $360^\circ$ ,  $540^\circ$ ,  $720^\circ$ . 6b)  $(n-2)180^\circ$ . 6c) Same result. 7a)  $3 \times 6$ ,  $4 \times 4$ . 7b) For sides  $x$  and  $y$ ,  $y = \frac{2x}{x-2}$ . 7c)  $3 \times 7 \times 42$ ,  $3 \times 8 \times 24$ ,  $3 \times 9 \times 18$ ,  $3 \times 10 \times 15$ ,  $3 \times 12 \times 12$ ,  $4 \times 5 \times 20$ ,  $4 \times 6 \times 12$ ,  $4 \times 8 \times 8$ ,  $5 \times 5 \times 10$ ,  $6 \times 6 \times 6$ . 9. Area of semicircle:  $(AB/2)^2 \pi/2 = AB^2 \pi/8$ . Area of quarter circle:  $(AB/\sqrt{2})^2 \pi/4 = AB^2 \pi/8$ . 11. Use angles to show similar triangles. By similarity  $\frac{c}{a} = \frac{a}{y}$ ,  $\frac{c}{b} = \frac{b}{x}$ . 13b) Cone:  $\int_0^h \pi(xr/h)^2 dx = \pi hr^2/3$ , cylinder:  $\int_0^h \pi r^2 dx = \pi hr^2$ . 13c) Radius:  $r(i) = (i/n)(r/h)$ , volume:  $\pi(h/n)(r(i))^2$ .

### Section 1.2

- 1a) 1, 2, 3, 9, 10, 11, 12, 22, 23, 31, 42, 44, 45, 46. 1b) 4, 8, 26, 34. 1c) 5, 6, 13, 15, 32–38, 43, 47, 48. 1d) 27–31, 39, 40. 3a) As  $\triangle ABD$  is similar to  $\triangle CDB$ ,  $\frac{1}{CD} = \frac{CD}{x}$ . So  $CD = \sqrt{x}$ .  
3d) Use part (a) with  $DB = \sqrt{x}$ ,  $\sqrt{\sqrt{x}}$ . 3e) Use part (a) to construct length  $\sqrt{2}$  and then length  $\sqrt{1 + \sqrt{2}}$ . 5a) Find the intersections of the perpendicular bisectors of the sides. 5b) Find the intersections of the angle bisectors. 11a) A square and a rhombus. 11b) Prove that both sets of two triangles formed by the diagonals are congruent using SSS. 11c) AAAS does not work: Consider two rectangles with the same height and different widths. 11d) SASAA and SASSS are sufficient, SAASA is not. 13b) Use addition formulas for  $\sin(a+b)$  and  $\cos(a+b)$ .

### Section 1.3

- 1a) Three points, three lines. 1c) Three points, three lines. 5a) I-1, I-2, I-3 (first part), II-1, II-2, II-3, III-1, III-2, III-3, V-1, V-2. 5e) V-2 (or I-1, I-2, I-3, II-1, II-2, II-4, and IV-1).

### Section 1.4

- 1a) III-1, III-2, III-3, III-4. 1b) Squares whose diagonals are vertical and horizontal; zero, one, or two points or one or two line segments. 1c) no. 3a) Three points, two lines; both lines are on all the points. 6a) An infinite checkerboard. 6b) A cube. 6c) A checkerboard on a doughnut.

**Section 1.5**

1.  $AD = AB^2/AC$ . 2a)  $x/y = \sqrt{2}$ . 2b)  $x/y = \sqrt{n}$ . 2c)  $x/y = (1 + \sqrt{5})/2$ .  
 4d)  $y = \sin(x) \sim y = \frac{1}{k} \sin(kx)$ . 13b) Use integration by substitution where  $u = \frac{x}{k}$ .  
 13c) A similar volume is  $k^3$  times the original volume.

**Section 1.6**

- 3a)  $0.5, \sqrt{2}/2, \sqrt{3}/2$ . 3b)  $2/(\pi\sqrt{3}) \approx 37\%$ . 3c) A tetrahedron, volume  $1/3$ . 5b) The distance between  $(1, b, 0)$  and  $(-1, b, 0)$  is 2. By Problem 2, the distance between  $(1, b, 0)$  and  $(0, 1, b)$  is  $\sqrt{1 + (b-1)^2 + b^2}$ . Set these equal to find  $b = (1 + \sqrt{5})/2$ . 6a)  $AC = (1 + \sqrt{5})/2$ ,  $BC = \sqrt{5 + \sqrt{5}}/\sqrt{2} \approx 1.902$ . 6b)  $2/BC \approx 1.05$ . 9b)  $195^\circ$ . 9c)  $\triangle ABN$  is  $1/8$  of sphere, so  $\triangle ACF$  is  $1/48$  of sphere. 13a) 42. 13b) 92. 13d)  $720/(10n^2 + 2)$ ,  $n = 11$ .  
 15a)  $2\pi R \sin(r/R)$ . 15b) Use  $f(x) = \sqrt{R^2 - x^2}$ ,  $a = R \cos(r/R)$ ,  $b = R$ .

**CHAPTER 2****Section 2.1**

2. If the vertices are  $(0, 0)$ ,  $(0, a)$ ,  $(b, c)$ , and  $(d, e)$ , then the midpoints are  $(0, \frac{a}{2})$ ,  $(\frac{b}{2}, \frac{a+c}{2})$ ,  $(\frac{b+d}{2}, \frac{c+e}{2})$ , and  $(\frac{d}{2}, 0)$ . Find the lengths of opposite sides. 3. If the vertices are  $C = (0, 0)$ ,  $(0, d)$ , and  $(e, f)$ , then  $\cos C = e/\sqrt{e^2 + f^2}$ . 6. Distance formula:  $\sqrt{k^2(x_1 - x_2)^2 + j^2(y_1 - y_2)^2}$ , circle:  $((x - a)/k)^2 + ((y - b)/j)^2 = r^2$ . 9a)  $y = x^2$ ,  $y = x - x^2$ ,  $y = x^4$ ,  $y = e^x$ ,  $y = \ln x$ . 9b) convex functions:  $y = x^2$ ,  $y = x^4$ ,  $y = e^x$ , concave functions:  $y = x - x^2$ ,  $y = \ln x$ . 9c) Second derivative of convex (concave) function is positive (negative). 10b) They are reflections in the real axis. 10c) The distance a point is from the origin. 11a) The product is the square of the modulus of that number. 11b) Change scale (similarity). 11c) Rotation around the origin.

**Section 2.2**

- 1a) The line that is the perpendicular bisector of the two points. 1b) A circle on the sphere. 1c) The plane that is the perpendicular bisector of the two points. Parts (a) and (b) are the intersections of this plane with their respective domains. 1d) A point, two points, and a line perpendicular to the plane determined by the three points. 3a) Parabola. 4b)  $xy - 1 = 0$ . 7a) Parabola. 7b) Ellipse. 7c) Degenerate ellipse (one point). 7d) Hyperbola. 7e) Ellipse.

**Section 2.3**

- 1a)  $\cos^2 t + \sin^2 t = 1$ .  $C$  and  $C^*$  are the same set of points— $C^*$  moves around the circle twice as fast. 1d) They give the same graph. 3a)  $(0, 0)$ ,  $(2, 0)$ . No instantaneous velocity at the bottom of the wheel, top of the wheel is going twice as fast as the train. 3b)  $(t - k \sin t, 1 - k \cos t)$ ,  $(1 - k \cos t, k \sin t)$ ,  $(1 - k, 0)$ . 5a)  $x = r \cos \theta$ ,  $y = r \sin \theta$ ,  $r = \sqrt{x^2 + y^2}$ ,  $\theta = \tan^{-1}(y/x)$ . 5b)  $x = f(\theta) \cos \theta$ ,  $y = f(\theta) \sin \theta$ . 5c)  $x^2 + y^2 - x = 0$ . 5d)  $y = 2 - x$ ,  $r = 2 \sec(\theta - 210^\circ)$ . 9. All hold except II-4, III-1, V-1, and V-2. 11a)  $x + y + 1 = 0$ ,  $\sqrt{2}x + y = 0$ . 11b)  $nx + y = 0$ ,  $\sqrt{2}x + y = 0$ . 11c) I-1, I-3, and IV-1.

**Section 2.4**

1.  $-0.15 + 3.161x + 11.227x^2 + 2.804x^3$ ,  $-97.4x^2 - 616x^3$ . 3a)  $x = t - 1$ ,  $y = -2 + 2t + 4t^2 - 3t^3$ . 3b)  $x = t + 1$ ,  $y = -1 + 4t^2 - 3t^3$ . 3c)  $x = t$ ,  $y = 1 + t - 8t^2 + 5t^3$ . 3d)  $x = t - 1$ ,  $y = 2 - 2t - 4t^2 + 3t^3$ . 3e)  $x = -t + 1$ ,  $y = 2 - 2t - 4t^2 + 3t^3$ . 5. Let  $f(t) = 1 + t^2$ ,  $g(t) = 2t + 4t^2 - 4t^3$ . Use eight Hermite curves  $(x_i(t), y_i(t))$ , where  $f = x_1 = y_2 = y_3 = x_8$ ,  $g = y_1 = x_2 = y_4 = x_7$ ,  $-f = x_4 = x_5 = y_6 = y_7$ , and  $-g = x_3 = y_5 = x_6 = y_8$ . (Other  $f$  and  $g$  will work.) 7a) Fifth degree. 7b)  $1 + x + 0.5x^3 - 0.5x^5$ .

- 7c)  $2n - 1$  degree. 9a)  $x - x^3/3! + x^5/5! - x^7/7!$ . 9b)  $8/3(x/\pi) - (8/3)(x/\pi)^3$ . 9c) From left to right:  $y = 0.858 + 2.639x + 1.101x^2 + 0.111x^3$ ,  $y = x + 0.057x^2 - 0.111x^3$ ,  $y = -x - 0.057x^2 + 0.111x^3$ , and  $y = -0.858 - 2.639x - 1.101x^2 - 0.111x^3$ .

**Section 2.5**

- 1a)  $\alpha(1, 2, 4) + \beta(2, 0, 0)$ ,  $2y - z = 0$ . 1b) Use the dot product of  $(2, 3, -4)$  and any  $(x, y, z)$  in the plane. Yes. 3b) Neither. 3c) Cut off one corner of a cube. 9a) Let  $e/v$  represent edges per vertex, and so on.

Dimension	1	2	3	4
$e/v$	1	2	3	4
$f/v$	0	1	3	6
$c/v$	0	0	1	4
$v$	2	4	8	16
$e$	1	4	12	32
$f$	0	1	6	24
$c$	0	0	1	8

**CHAPTER 3****Section 3.1**

- 2a) The centers of the two circles and one of their points of intersection,  $(0.8, 0.6)$ , form a right triangle. 2b)  $(0.65, \pm 0.45)$ . 2d) Angle at  $(0, 0)$  is  $69.4^\circ$ , at  $(0.65, \pm 0.45)$  is  $18.4^\circ$ . 3. II-4, III-4, and IV-1. 5.  $(1, 1)$ ,  $(4, 2)$ , and  $(4, \sqrt{10})$ ;  $8.13^\circ$ ,  $63.43^\circ$ , and  $90^\circ$ .

**Section 3.2**

1. All can occur. 8. Suppose in omega triangles  $\triangle AB\Omega$  and  $\triangle CD\Lambda$  that  $\angle AB\Omega \cong \angle CDA$  and  $\angle BA\Omega \cong \angle DC\Lambda$ . For a contradiction suppose that  $AB$  is longer than  $CD$ . Construct  $B'$  between  $A$  and  $B$  with  $AB' \cong CD$ . Use Theorem 3.2.7 on  $\triangle AB'\Omega$  and  $\triangle CD\Lambda$  and then use Theorem 3.2.6 for  $\triangle BB'\Omega$ .

**Section 3.3**

2. Assume that  $C'$  is on  $\overrightarrow{DC}$  with  $DC' \cong BA$  so that  $ABDC'$  is a Saccheri quadrilateral. Theorem 3.3.1 and Corollary 3.2.1 show  $AC'$  is ultraparallel to  $BD$ . Show  $C$  is between  $D$  and  $C'$ .

**Section 3.4**

3. The maximum defect a triangle can have is  $180^\circ$ ,  $K = k \times 180^\circ$ . 4. The area of  $\triangle AB_1B_i$  is finite even as  $i \rightarrow \infty$ , so the area of  $\triangle AB_iB_{i+1}$  approaches 0 as  $i \rightarrow \infty$ . 6.  $K_n = (n - 2)K$ .

**Section 3.5**

- 2a) Consider two doubly right triangles with different included sides. 3. In spherical geometry, all lines perpendicular to a given line intersect in two opposite points. 6a) Half of a sphere,  $2\pi r^2$ .

**CHAPTER 4****Section 4.1**

- 1a) Both formulas clearly give functions. To show one-to-one and onto, solve  $y = \alpha(x)$  and  $y = \beta(x)$  for  $x$  to see that the choice of  $x$  in each is unique. 1b)  $(2x - 1)^3$  and  $2x^3 - 1$ .

**1c)**  $\sqrt[3]{x}$  and  $0.5x + 0.5$ . **1d)**  $\sqrt[3]{0.5x + 0.5}$  and  $0.5\sqrt[3]{x} + 0.5$ ,  $\beta^{-1} \circ \alpha^{-1}$ . **2a)** Rotation of  $180^\circ$  around the fixed point  $(1, 2)$ . All lines through  $(1, 2)$  are stable. **2b)** Mirror reflection over the line  $y = x + 1$ , which are the fixed points. The other stable lines are  $y = -x + c$ . **2c)** Dilation by a factor of 2 about the fixed point  $(0, 0)$ . All lines through  $(0, 0)$  are stable. **2d)** Dilation by a factor of 0.5 about the fixed point  $(2, -2)$ . All lines through  $(2, -2)$  are stable.

#### Section 4.2

**1.**  $(-1, 1), (1, -1), (y - 1, -x + 1)$ . **7.** Note that  $d(P, Q) + d(Q, R) = d(P, R)$  just when  $Q$  is between  $P$  and  $R$ . **8.** Use Theorem 4.2.7 and the fact that the composition is direct. It is a translation when the angles of rotation are opposite. **11.**  $V$  consists of translations,  $180^\circ$  rotations, vertical and horizontal mirror and glide reflections. Proof. Let  $l$  be a vertical line.  $\iota(l) = l$  is vertical, so  $\iota \in V$ . If  $\alpha, \beta \in V$ ,  $\beta(l)$  and so  $\alpha(\beta(l))$  are vertical, so  $\alpha \circ \beta \in V$ . Suppose  $\alpha \in V$  and  $k$  the vertical line so that  $\alpha(k) = l$ . Then  $\alpha^{-1}(l) = k$ , so  $\alpha^{-1} \in V$ .

#### Section 4.3

**1.**  $A$  is a rotation around  $(1, 3, 1)$  with no stable lines.  $B$  is a translation with no fixed points and stable lines  $y = -3.5x + c$ .  $C$  is a mirror reflection over  $y = x/3 + 5/9$ , the fixed points. The other stable lines are  $y = -3x + c$ .  $D$  is a glide reflection over the stable line  $y = x/3 + 1/6$ .  $E$  is a rotation around  $(1, 1 + \sqrt{2}, 1)$  with no stable lines.

$$\begin{array}{l} \text{3a)} \begin{bmatrix} \cos 30 & -\sin 30 & 3.5 - \sqrt{3} \\ \sin 30 & \cos 30 & 2 - 1.5\sqrt{3} \\ 0 & 0 & 1 \end{bmatrix}. \quad \text{3b)} \begin{bmatrix} -0.6 & 0.8 & 0 \\ 0.8 & 0.6 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad \text{3c)} \begin{bmatrix} -0.6 & 0.8 & -0.8 \\ 0.8 & 0.6 & 0.4 \\ 0 & 0 & 1 \end{bmatrix}. \end{array}$$

$$\begin{array}{l} \text{3d)} \begin{bmatrix} -0.6 & 0.8 & c \\ 0.8 & 0.6 & 2c \\ 0 & 0 & 1 \end{bmatrix}. \quad \text{8a)} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad \text{8b)} \begin{bmatrix} -1 & 0 & 2u \\ 0 & -1 & 2v \\ 0 & 0 & 1 \end{bmatrix}. \quad \text{8c)} \text{ Direct isometry,} \end{array}$$

rotation of  $180^\circ$ . **8d)** Lines through  $(u, v, 1)$ . **8e)** Translations twice the distance between the centers. You get the inverse translation.

#### Section 4.4

**1a)**  $(1, 0, 1), (0, 2, 1), (-4, 0, 1)$ , and  $(0, -8, 1)$ . Spiral. **1b)**  $(1, 1, 1), (-2, 2, 1)$ , and  $(-4, -4, 1)$ . Yes. **1c)**  $M$  is a rotation by  $\theta$  and a scaling by  $r$ ,  $S$  is a rotation by  $\theta/2$  and a scaling by  $\sqrt{r}$ . **1d)** For  $C$  use a rotation by  $\theta/3$  and a scaling by  $\sqrt[3]{r}$  and for  $N$  use a rotation by  $\theta/n$  and a scaling by  $\sqrt[n]{r}$ . **11c)** This IFS fractal is the part of Fig. 4.25 on the  $x$ -axis. **12.** The four corners of the square,  $(0, 0, 1), (1, 0, 1), (0, 1, 1)$ , and  $(1, 1, 1)$ , must go to points in that square. For

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}, \text{ the following must be between 0 and 1: } c, a+c, b+c, a+b+c, f, d+f, e+f, \text{ and } d+e+f.$$

#### Section 4.5

**1b)**  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ , rotation of  $180^\circ$  around the  $z$ -axis. The other of

these two matrices. **1d)** Rotations of  $120^\circ$  around opposite vertices of the cube. **1e)** For example,

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$  is a  $90^\circ$  rotation around the  $x$ -axis and  $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$  is a  $120^\circ$  rotation

**1c)**  $\sqrt[3]{x}$  and  $0.5x + 0.5$ . **1d)**  $\sqrt[3]{0.5x + 0.5}$  and  $0.5\sqrt[3]{x} + 0.5$ ,  $\beta^{-1} \circ \alpha^{-1}$ . **2a)** Rotation of  $180^\circ$  around the axis through  $(1, 1, 1, 1)$  and  $(-1, -1, -1, 1)$ . **1f)**  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ .

**2a)**  $\begin{bmatrix} -1 & 0 & 0 & a \\ 0 & -1 & 0 & b \\ 0 & 0 & -1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$ , rotatory reflection. **2b)** Translation twice the distance between the centers. **3.**  $\begin{bmatrix} \cos \theta & 0 & -\sin \theta & 0 \\ 0 & 1 & 0 & k \\ \sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$  (or with the  $-$  on the other  $\sin \theta$ , depending on the orientation of the axes).

#### Section 4.6

**1a)** Center  $(0, 0)$  and radius 2, center  $(-1, 0)$  and radius  $\sqrt{10}$ . **1b)** All  $x$ -axis except points between  $x = 1$  and  $x = 4$ . **1c)**  $r = \sqrt{c^2 - 5c + 4}$ , where  $c$  is the  $x$ -coordinate of the center. **5.** No, because the inversion of at least some of the sides are circles. **6a)** Translation. **6b)** If  $a > 0$ , a dilation by a ratio of  $a$ . If  $a < 0$ , also a rotation of  $180^\circ$ . A rotation. A composition of a dilation and a rotation. **6c)** A mirror reflection over the real axis. **7a)**  $2/(z + 2i) - 2i$ . **7b)**  $4/(z + 3i) - 3i$ . **7c)**  $E$  has center  $-1.5i$  and radius 0.5. **7e)**  $-4i/3$ . **7f)**  $v_C \circ v_D$ . **9b)** The equator. Mirror reflection over the equator.

## CHAPTER 5

#### Section 5.1

**3.** Iranian: rotations of  $0^\circ, 120^\circ$ , and  $240^\circ$  and three mirror reflections. Byzantine: rotations of  $0^\circ, 90^\circ, 180^\circ$ , and  $270^\circ$  and four mirror reflections. Afghani: rotations of  $0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ$ , and  $300^\circ$ . **5a)** 25%, 25%. **5b)** 33.3%, 33.3%. **5c)** 22.2%, 11.1%. **7a)** Mexican: translations,  $180^\circ$  rotations, vertical mirror reflections, and horizontal glide reflections. Chinese: translations,  $180^\circ$  rotations, vertical and horizontal mirror reflections, and horizontal glide reflections.

#### Section 5.2

**1a)** Gothic:  $C_3$ ; Islamic:  $D_{10}$ ; Gothic:  $C_2$ . **1b)** General:  $C_1$ ; parallelogram:  $C_2$ ; kite and isosceles trapezoid:  $D_1$ ; rectangle and rhombus:  $D_2$ ; and square:  $D_4$ . **3a)** Rotations of  $0^\circ, 90^\circ, 180^\circ$ , and  $270^\circ$ . Yes,  $C_4$ . **3b)** Rotations of  $45^\circ, 135^\circ, 225^\circ$ , and  $315^\circ$ . No. **3c)** Yes,  $C_8$ . **3e)** Color preserving: Rotations of  $0^\circ, 120^\circ$ , and  $240^\circ$ , yes,  $C_3$ ; Color switching: Rotations of  $40^\circ, 80^\circ, 160^\circ, 200^\circ, 280^\circ$ , and  $320^\circ$ , no; Union: Yes,  $C_9$ . **6a)** Cube: 48; tetrahedron: 24; octahedron: 48; dodecahedron: 120; and icosahedron: 120. **6b)** Triangular prism: 12; square prism: 16. **6c)** 4n. **6d)** Yes, four of the cube's vertices form a tetrahedron.

#### Section 5.3

**1a)** pmm2. **1b)** p112. **1c)** p1g1. **1d)** pmg2. **1e)** p1m1. **1f)** pm11. **3a)** pg. **3b)** p2. **3c)** p3. **3d)** cm. **3e)** p6m. **3f)** cmm. **11.** p4g, pgg.

#### Section 5.4

**1.** 8, 16, 48. Rotations of  $180^\circ$  around the  $x$ -,  $y$ -, and  $z$ -axes, mirror reflections over the  $xy$ -,  $xz$ -, and  $yz$ -planes, the identity and the central symmetry. (See Problem 2 of Section 4.5.) **6a)** Rotations of  $90^\circ, 180^\circ, 270^\circ$ , and  $0^\circ$  around the centers of opposite faces, rotations of  $120^\circ$

and  $240^\circ$  around opposite vertices, and rotations of  $180^\circ$  around centers of opposite edges; mirror reflections over six planes through opposite edges and three planes between opposite faces; and 15 rotatory reflections. **6b)** Rotations of  $120^\circ$ ,  $240^\circ$ , and  $0^\circ$  around a vertex and the center of the opposite face and rotations of  $180^\circ$  around centers of opposite edges; mirror reflections over six planes through an edge and the center of the opposite edge; and six rotatory reflections. **6c)** For icosahedron: rotations of multiples of  $72^\circ$  around opposite vertices, rotations of  $120^\circ$  and  $240^\circ$  around centers of opposite faces, and rotations of  $180^\circ$  around centers of opposite edges; mirror reflections over 15 planes through opposite edges; 45 rotatory reflections.

### Section 5.5

- 1a)**  $D_2$ . **1b)**  $D_1$ ,  $D_1$ , and  $C_2$ . **2a)**  $D_6$ . **2b)**  $D_1$ ,  $D_1$ , and  $D_2$ . **2c)**  $D_2/D_1$ ,  $D_1/C_1$ , and  $D_6/D_3$ .  
**5.** p31m. **6a)** 0.9945. **6b)** 0.2273. **6c)** 3, 5, and 7. **9a)** Hyperbola. **9b)** Hyperbola.  
**9c)** Asymptotes for parts (a) and (b).

### Section 5.6

- 2a)**  $\ln 2/\ln 2 = 1$ . **2b)**  $\ln 3/\ln 2 \approx 1.585$  **2c)**  $\ln 2/\ln 3 \approx 0.631$ . **2d)**  $\ln 5/\ln 3 \approx 1.465$ .  
**2e)**  $\ln 8/\ln 4 = 1.5$ . **2f)**  $\ln 3/\ln 2 \approx 1.585$ . **4a)**  $\ln 6/\ln 2 \approx 2.585$ . **4b)**  $\ln 13/\ln 3 \approx 2.335$ .  
**4c)**  $\ln 26/\ln 3 \approx 2.966$ . **5a)**  $1/9 + 5/(9^2) + 5^2/(9^3) + \dots = 1/4$ .  
**5b)**  $1/27 + 13/(27^2) + 13^2/(27^3) + \dots = 1/14$ . **5c)** 0.

## CHAPTER 6

### Section 6.1

- 5. 0.6.** **11a)**  $V$ ,  $E$ , and  $D$ . **11b)**  $\Delta BEV$ ,  $\Delta CFU$ ,  $W$  and  $P$ ,  $A$ , and  $D$ .

### Section 6.2

- 1b)**  $H(PQ, SR)$ ,  $H(QP, RS)$ ,  $H(QP, SR)$ ,  $H(RS, PQ)$ ,  $H(RS, QR)$ ,  $H(SR, PQ)$ , and  $H(SR, QR)$ . **2.**  $j/2^k$ , where  $j$  and  $k$  are positive integers. **6b)** For example, (vi) becomes: If  $pq//rs$ , then  $p$ ,  $q$ ,  $r$ , and  $s$  are distinct, concurrent lines,  $pq//sr$  and  $rs//pq$ . **10a)** Yes, yes. **10b)** Yes.

### Section 6.3

- 1a)**  $(1, m, 0)$ , parallel lines “meet” at infinity. **2.**  $X_a = (a, 0, 1)$ ,  $Y_b = (0, b, 1)$ ,  $P_{ab} = (a, b, 1)$ , and  $(a, b, 0)$ . **5a)** One of the orderings in (b). **5b)** The possible orderings of  $P$ ,  $Q$ ,  $R$ ,  $S$ , and  $T$  are  $P, S, Q, R, T$ ;  $P, T, R, Q, S$ ;  $Q, R, T, P, S$ ;  $Q, S, P, T, R$ ;  $R, Q, S, P, T$ ; and the reverse of these. **6b)**  $1/r$ ,  $r/(r-1)$ ,  $(r-1)/r$ ,  $1-r$ , and  $1/(1-r)$ .  
 $R(A, B, C, D) = R(B, A, D, C) = R(C, D, A, B) = R(D, C, B, A)$ . **6c)** Yes. **6d)** 0, 1, and  $\infty$ . **7a)**  $-xz + xy + yz = 0$ ,  $y - z = 0$ , and  $x + z = 0$ .  $(1, 0, 0)$  and  $(0, 1, 0)$ . **7b)**  $x + y = 0$ ,  $x - y = 0$ , and  $x^2 - y^2 - z^2 = 0$ .  $(1, -1, 0)$  and  $(1, 1, 0)$ . **7c)**  $x^2 - xz - yz = 0$ .  $(0, 1, 0)$ . **9b)** In nonhomogeneous coordinates:  $y = 1$ ,  $y = -1$ ,  $x = 1$ ,  $x = -1$ ,  $y = x$ , and  $y = -x$ . **9e)** Unit circle, ellipses, and hyperbolas.

### Section 6.4

- 1a)**  $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ , where  $a \neq 0$ ;  $\begin{bmatrix} a & 0 \\ c & d \end{bmatrix}$ , where  $d \neq 0$ ; and  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , where  $a + b = c + d$ .  
**1b)**  $\begin{bmatrix} a & b & c \\ 0 & e & f \\ 0 & h & i \end{bmatrix}$ , where  $a \neq 0$ ;  $\begin{bmatrix} a & 0 & c \\ d & e & f \\ g & 0 & i \end{bmatrix}$ , where  $e \neq 0$ ;  $\begin{bmatrix} a & b & 0 \\ d & e & 0 \\ g & h & i \end{bmatrix}$ , where  $i \neq 0$ ; and

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, \text{ where } a + b + c = d + e + f = g + h + i. \quad \text{2a)} \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}. \quad \text{2b)} \begin{bmatrix} -1 & 0 \\ -3 & 2 \end{bmatrix}.$$

$$\text{6a)} (1, -2, 1), (4, -3, 1), \text{ and } (-8, 1, 1) \text{ are on } 3y + x + 5z = 0. \quad \text{6b)} \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ -1 & -3 & 7 \end{bmatrix}.$$

$$\text{7a)} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -0.5 & 1 \\ 0 & -0.5 & -1 \end{bmatrix}, 4x^2 - 2yz = 0. \quad \text{7c)} y = 2x^2, \text{ a parabola.} \quad \text{9a)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -w & 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & -w \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ and } (1+2w)x^2 + y^2 - 2xz = 0. \quad \text{9b)} 2x^2 + y^2 - 2x = 0, 0.5x^2 + y^2 - 2x = 0,$$

$2x = y^2$ , and  $-x^2 + y^2 - 2x = 0$ . **9d)** For  $w > -0.5$  and  $w \neq 0$ , the image is an ellipse. For  $w < -0.5$ , the image is a hyperbola. For  $w = -0.5$ , the image is a parabola. **9f)** For  $w = 0.5$ ,  $[2, 0, -2]$  and  $[0.5, 1, -1]$ . For  $w = -0.25$ ,  $[0.5, 0, -2]$  and  $[-0.25, 1, -1]$ . For  $w = -0.5$ ,  $[0, 0, -2]$  and  $[-0.5, 1, -1]$ . For  $w = -1$ ,  $[-1, 0, -2]$  and  $[-1, 1, -1]$ .

### Section 6.5

$$\text{4. } Y_b = \begin{bmatrix} \sqrt{1-b^2} & 0 & 0 \\ 0 & 1 & b \\ 0 & b & 1 \end{bmatrix}. \text{ Transposes of each other. Use } a = c \text{ and } b = d/\sqrt{1-c^2}.$$

### Section 6.6

- 1.** Four distinct points  $(a, b, c, d)$ ,  $(e, f, g, h)$ ,  $(i, j, k, l)$ , and  $(m, n, o, p)$ , in  $\mathbb{P}^3$  are coplanar iff the determinant  $\begin{vmatrix} a & e & i & m \\ b & f & j & n \\ c & g & k & o \\ d & h & l & p \end{vmatrix}$  is 0. **2a)**  $n \leq 3$ . **2b)**  $n \leq 4$ . **2c)**  $n \leq k + j - 2$ . **2d)** Its minimum dimension increases.

## CHAPTER 7

### Section 7.1

- |                  |                  |                  |                  |
|------------------|------------------|------------------|------------------|
| <b>1a)</b> $C_2$ | <b>1b)</b> $C_2$ | <b>1c)</b> $C_2$ | <b>1d)</b> $C_2$ |
| $A_1$            | $B_2$            | $C_3$            | $A_1$            |
| $B_3$            | $C_1$            | $A_2$            | $B_4$            |
- 2a)** Shift 2 to the right and 1 down. Shift 1 right and 2 down. **3.** 1 2 3, 4 5 6, 7 8 9; 1 4 7, 2 5 8, 3 6 9; 1 6 8, 2 4 9, 3 5 7; and 1 5 9, 2 6 7, 3 4 8. **5b)**  $4k + 1$ . **7a)** A pentagon, five points. **7c)**  $k^2 + 1$ .

### Section 7.2

- 4a)** 4, 4. **8a)** (ii). **8c)** Theorem 7.2.1 (i), (ii), (iii), and (iv). **9a)** A triangle. **9c)** Four points with three points on one line and all other lines having two points.

### Section 7.3

- 1a)**  $b = \lambda(v/k)(v-1)/(k-1)$ . **2a)**  $r \geq k$ . **2b)**  $r = k = n+1$ ;  $v = b = k^2 - k + 1 = n^2 + n + 1$ ;  $n = k - 1$  is the order of the projective plane. **5a)** One line is 1, 2, 5, and 7. Rotate these numbers for the other lines. **7a)** 1 vs. 2 and 3 vs. 4; 1 vs. 3 and 2 vs. 4; and 1 vs. 4 and 2 vs. 3. **7b)**  $2n - 1$ . **7c)** In the first round pair players that add to  $2n$ ,  $n$  sits out.

$$\begin{matrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{matrix}$$

Now rotate. With  $2n$  players,  $2n$  plays the person sitting out. **9a)**

$$\begin{matrix} 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{matrix}$$

**1a)**  $\begin{matrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{matrix}$  **1b)**  $\begin{matrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{matrix}$

**1c)** Switches  $x$ - and  $y$ -coordinates. Similar to mirror reflection.  
**1d)** Rotation of  $90^\circ$  around  $(1, 1, 1)$ . Rotation of  $180^\circ$  around  $(0, 0, 1)$ . Translation of 1 to the right and 1 down. **3.** Many answers possible for each part. **3a)**  $[2, 2, 2]$  has eight points.

**3b)**  $[2, 1, 0]$  and  $[0, 1, 0]$  have both  $(0, 0, 1)$  and  $(2, 0, 1)$  on them. **3c)**  $[1, 1, 0]$  and  $[1, 3, 1]$ .

**3d)** (i) and (iii). **3e)** For part (a),  $[2, 4, 2]$  has 12 points. For (b),  $[3, 1, 0]$  and  $[0, 1, 0]$ . For (c),  $[1, 1, 0]$  and  $[1, 4, 1]$ . For (d), (i) and (iii). **5a)**  $(2, 0, 1)$ . **5b)** Four points:  $(0, 0, 1)$ ,  $(0, 1, 1)$ ,  $(1, 0, 1)$ , and  $(1, 1, 1)$ . Six lines:  $[1, 0, 0]$ ,  $[0, 1, 0]$ ,  $[1, 1, 4]$ ,  $[1, 4, 0]$ ,  $[0, 1, 4]$ , and  $[1, 0, 4]$ .

**5c)**  $(3, 0, 1)$  is the only choice for  $S$ . **9.**  $x^2 + y^2 = 1$  has  $(0, 1, 1)$ ,  $(0, 4, 1)$ ,  $(1, 0, 1)$ , and  $(4, 0, 1)$ .  $x^2 + 4y = 0$  has  $(1, 1, 1)$ ,  $(4, 1, 1)$ ,  $(2, 4, 1)$ ,  $(3, 4, 1)$ , and  $(0, 0, 1)$ .  $x^2 + 3y^2 = 1$  has  $(1, 0, 1)$ ,  $(4, 0, 1)$ ,  $(2, 2, 1)$ ,  $(2, 3, 1)$ ,  $(3, 2, 1)$ , and  $(3, 3, 1)$ . In  $\mathbf{PZ}_5^2$ , they all have six points. The first has  $(1, 2, 0)$  and  $(1, 3, 0)$ , the second has  $(0, 1, 0)$ , and the third has no other points.

#### Section 7.4

**1a)**  $[1, 2, 1]$ . **1b)**  $(2, 0, 1)$ . **1c)** Switches  $x$ - and  $y$ -coordinates. Similar to mirror reflection.

**1d)** Rotation of  $90^\circ$  around  $(1, 1, 1)$ . Rotation of  $180^\circ$  around  $(0, 0, 1)$ . Translation of 1 to the right and 1 down. **3.** Many answers possible for each part. **3a)**  $[2, 2, 2]$  has eight points.

**3b)**  $[2, 1, 0]$  and  $[0, 1, 0]$  have both  $(0, 0, 1)$  and  $(2, 0, 1)$  on them. **3c)**  $[1, 1, 0]$  and  $[1, 3, 1]$ .

**3d)** (i) and (iii). **3e)** For part (a),  $[2, 4, 2]$  has 12 points. For (b),  $[3, 1, 0]$  and  $[0, 1, 0]$ . For (c),  $[1, 1, 0]$  and  $[1, 4, 1]$ . For (d), (i) and (iii). **5a)**  $(2, 0, 1)$ . **5b)** Four points:  $(0, 0, 1)$ ,  $(0, 1, 1)$ ,  $(1, 0, 1)$ , and  $(1, 1, 1)$ . Six lines:  $[1, 0, 0]$ ,  $[0, 1, 0]$ ,  $[1, 1, 4]$ ,  $[1, 4, 0]$ ,  $[0, 1, 4]$ , and  $[1, 0, 4]$ .

**5c)**  $(3, 0, 1)$  is the only choice for  $S$ . **9.**  $x^2 + y^2 = 1$  has  $(0, 1, 1)$ ,  $(0, 4, 1)$ ,  $(1, 0, 1)$ , and  $(4, 0, 1)$ .  $x^2 + 4y = 0$  has  $(1, 1, 1)$ ,  $(4, 1, 1)$ ,  $(2, 4, 1)$ ,  $(3, 4, 1)$ , and  $(0, 0, 1)$ .  $x^2 + 3y^2 = 1$  has  $(1, 0, 1)$ ,  $(4, 0, 1)$ ,  $(2, 2, 1)$ ,  $(2, 3, 1)$ ,  $(3, 2, 1)$ , and  $(3, 3, 1)$ . In  $\mathbf{PZ}_5^2$ , they all have six points. The first has  $(1, 2, 0)$  and  $(1, 3, 0)$ , the second has  $(0, 1, 0)$ , and the third has no other points.