HEAT TRANSFER

We want to investigate how heat gets from one place to another. (GIVE SOME EXAMPLES) In general, three methods:

1. conduction -- heat moves through matter without bulk motion

2. convection -- bulk motion of matter transfers heat

3. radiation -- infra-red electromagnetic radiation, i.e., em radiation with wavelengths > 700 nm (7000 Å); thus, transfer of heat is independent of matter

Note that here we are talking about non-equilibrium processes! give a few examples

CONVECTION I will leave to you and the text -- tends to be complicated, obeys no simple law--discuss

CONDUCTION

Consider for example a metal bar whose ends are held at different temperatures

\[ P = \frac{dQ}{dt} = -kA \frac{dT}{dx} \]

where
A = cross-sectional area of the bar
k = thermal conductivity UNITS: J/m-s-K

EXERCISE: confirm units

note: - sign ==> heat flows from hot to cold
SPECIAL CASE: if $dT/dx = \text{constant}$ (discuss physical significance)

Then

$$P = \frac{dQ}{dt} = -kA \frac{\Delta T}{\Delta x} = -kA \frac{\Delta T}{L} = kA \frac{T_H - T_C}{L}$$

where $L$ is the length of the conducting bar and $k$ is the thermal conductivity—Discuss. Compare HRW Eq. 19-32; note HRW uses just last form, so no minus sign.

Exercise: show $[k] = \frac{\text{Watts}}{\text{m-K}}$

and it is often convenient to define the

DEF: THERMAL RESISTANCE $R$ (HRW definition)

$$R_{\text{HRW}} = \frac{L}{k}$$

Does the def make sense? discuss intuitively

A better (and more common) definition is

$$R_{\text{better}} = \frac{L}{kA}$$

Our heat conduction equation can therefore be written

$$P = -\frac{kA}{L} \Delta T = \frac{1}{R_b} \Delta T$$

or

$$\Delta T = RP$$

Hereafter, assume $R = R_{\text{better}}$. Note and discuss analogy to Ohm's Law:

$$V = RI$$

Thus, one can think in terms of a temp difference "driving" a flow of heat, just as a potential difference “drives” a current!
Moreover, since these equations are formally the same, we can apply what we already know about Ohm’s law, series and parallel circuits, to heat conduction.

Make heat flow analogies to series, parallel circuits

EXAMPLE (HRW, p. 444)—Consider the following: Find the power being conducted in the following configuration. Assume the cross-sectional areas are the same, for simplicity.

\[ P = \frac{dQ}{dt} \]

\[ T_{\text{hot}} \quad k_1 \quad T_x \quad k_2 \quad T_{\text{cold}} \]

\[ L_1 \quad T_x \quad L_2 \]

\[ T_h > T_x > T_c \]

Note that for steady-state conduction, the power must be same everywhere—if it weren’t, what would happen??

Consequently

\[ \frac{T_h - T_x}{R_1} = P = \frac{T_x - T_c}{R_2} \]

But we can also talk about the effective thermal resistance (discuss):

\[ T_H - T_c = R_{\text{eff}} P \]

But note that

\[ T_H - T_c = T_H - T_x + T_X - T_c \]

\[ = R_1 P + R_2 P = (R_1 + R_2)P \]

Apparently, then,

\[ R_{\text{eff}} = R_1 + R_2 \]

That is, thermal resistances (my definition!) in series act like electrical resistances in series. We can now find the power transmitted:
\[ T_H - T_c = R_{\text{eff}} P = (R_1 + R_2) P \]

or, substituting,
\[ T_H - T_c = \left( \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} \right) P \]

or
\[ P = \frac{A(T_H - T_c)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}} \]

This result is the same as HRW Eq 19-36.

Exercise: Find \( T_x \).

Exercise: Generalize for different cross-sectional areas.

Exercise: Consider the following configuration:

Show that
\[ \frac{1}{R_{\text{eff}}} = \frac{1}{R_1} + \frac{1}{R_2} \]

Note the connection with insulation and “R-values”
\[ \mathcal{R} = R_{H_RW} = \frac{L}{k} \]

The units, unfortunately, are ft\(^2\) °F h/BTU
Stefan-Boltzmann Law of Radiation

If we heat up an object--say a piece of iron--it first glows red, then white, and finally (if it doesn't melt first) blue; even so, if we pass the light through a prism or a diffraction grating, we get the entire spectrum.

discuss, draw Planck's Law curve

Intuitively, the hotter the object is (compared to its surroundings), the more energy it gives off; it can be shown that the power--that is, the rate of energy loss--is given by the S-B law:

\[ P = \sigma \varepsilon A T^4 \]

where \( A \) is the surface area of the object;

\[ \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4, \text{ the Stefan-Boltzmann constant} \]

and \( \varepsilon \) is the emissivity, \( 0 \leq \varepsilon \leq 1 \).

The emissivity is of some interest; an object that is completely "black" entirely absorbs all incident radiation (\( \varepsilon = 1 \)); likewise, if \( \varepsilon = 0 \), the object reflects all incident radiation!

discuss, talk about practical consequences; discuss question 15

THERMAL ENERGY BALANCE

We frequently encounter systems that are not in equilibrium, but are nevertheless "balanced," in the sense that the energy coming in balances the energy going out:

that is, power in = power out

examples: house
         earth

discuss, do one or two examples

Example  Consider the earth’s energy balance:

- Absorbs energy from sun
- Radiates energy into space

If steady-state (constant temp of earth), power absorbed = power radiated
Consider the following order of magnitude calculation:

Energy absorbed from sun—at earth’s orbit \( S = 1.37 \, \text{kW/m}^2 \)

But about 30% is reflected back into space, so that \( S_{\text{eff}} = 960 \, \text{W/m}^2 \)

So if power in = power out,

\[
\pi R_e^2 S_{\text{eff}} = \sigma \varepsilon A T^4 = \sigma \varepsilon (4\pi R_e^2 T^4)
\]

if we take emissivity \( \varepsilon = 1 \), we have

\[
T = \left( \frac{S_{\text{eff}}}{4\sigma} \right) = 255 \, \text{K} = -18 \, ^\circ\text{C}
\]