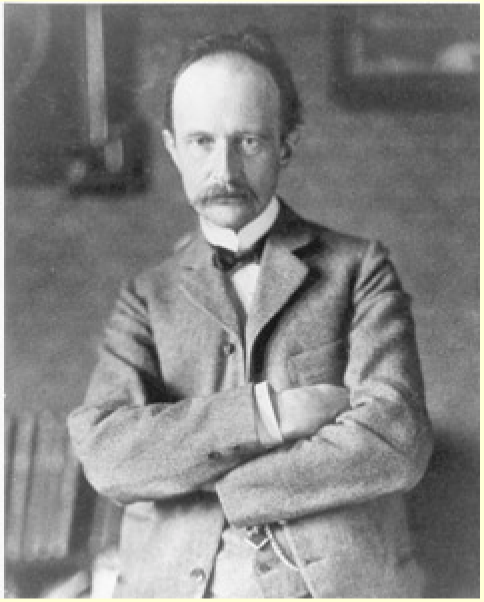
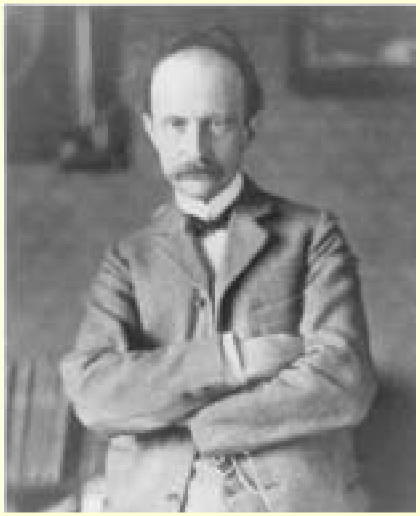
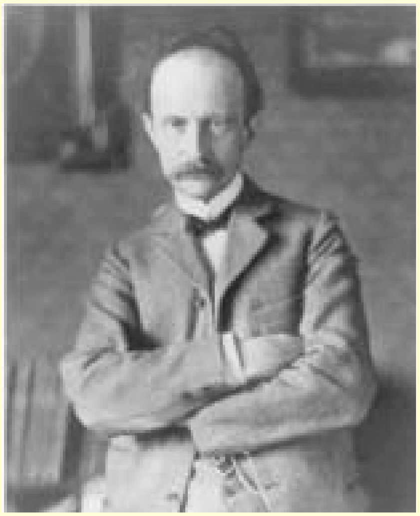


Max Planck
1858–1947



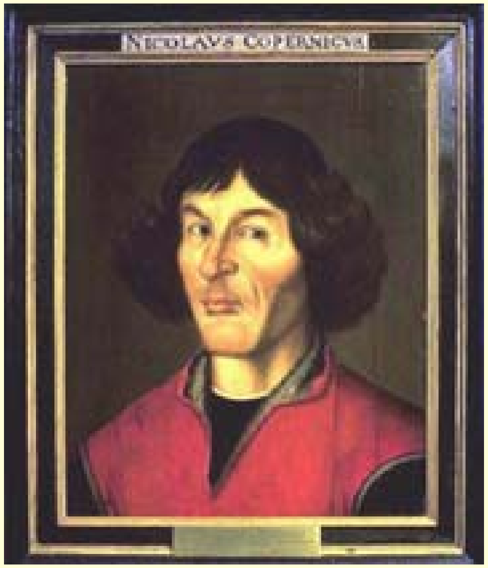
Planck



Planck

the Quantum

The Textbooks



In 1555 [sic!] Copernicus shewed that the observed motions of the sun and planets were far more simply explained by supposing ... that all the planets, including the earth, revolved around a fixed central sun.

Copernicus suggested that a simpler description of celestial motions could be given by assuming that the sun was at rest at the center of the universe.

The planetary positions predicted by Copernicus were *not* as accurate as those found using ... the more complicated geocentric theory of Ptolemy.

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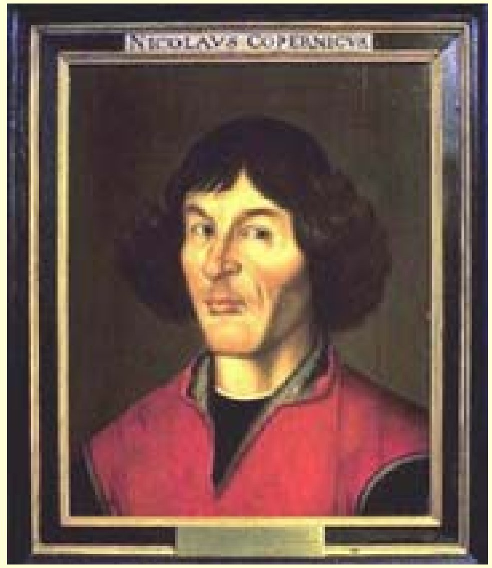


The Historians

It is sometimes claimed that Ptolemaic ... astronomy fails on the questions of accuracy and simplicity. Unfortunately, this explanation is completely false.

James Evans, *The History and Practice of Ancient Astronomy.*

Anyone who thinks that Copernican theory is "simpler" than Ptolemaic theory has never looked at Book III of *De revolutionibus.*

Swerdlow and Neugebauer, *Mathematical Astronomy in Copernicus's De Revolutionibus*

Revenge of the Physicists?

The Textbooks

Planck ... had to make a drastic, quite unjustified assumption: that the oscillators could only emit and absorb energy of frequency f in units of hf, where h is a new universal constant .... Planck called these energy units **quanta.**

Planck departed radically from classical physics. He postulated that the energies of the oscillators are quantized... the only permitted values of the energy are ...

$0, h\nu, 2h\nu, 3h\nu$. All other values of the energy are forbidden. The constant h is Planck's constant.

The Historians

[Planck] had taken a fateful step by requiring the energy of one of his oscillators always to be a discrete multiple of an energy unit.

I am convinced that, with Planck's particular sensitivity to the importance of the natural constants, it was these results that assured him that quanta were more than an ad hoc hypothesis, useful only for arriving at the radiation law.

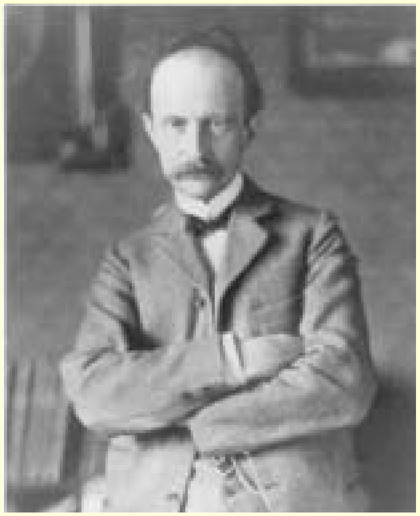
Reading his papers today, we are struck by how little emphasis he gave to this radical idea. …

Martin J. Klein

My point is not that Planck doubted the reality of quantization or that he regarded it as a formality to be eliminated during the further development of his theory. Rather, I am claiming that the concept of restricted resonator energy played no role in his thought.

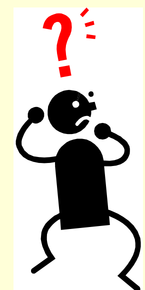
Planck's resonators ... absorbed and emitted radiation continuously.... His theory was still classical.

Thomas S. Kuhn



Planck

the Quantum



and

the Historians

Who was Max Planck?

- born 1858
- Ph.D. Munich, 1879
- “Privatdocent” in Munich until 1885
- 1885: appointed “extraordinary” Professor of theoretical physics in Kiel, in Northern Germany near the border with Denmark
- 1889: appointed extraordinary professor of theoretical physics, and director of an institute, in Berlin; became professor in 1892

Throughout this period, Planck’s research centered on the Second Law of Thermodynamics, first stated and then elaborated over the course of the 19th century.

Second Law of Thermodynamics

"Some Processes are Irreversible"
(Tom Moore)

Planck and the Second Law

Entropy:

A mathematical function that expresses how “irreversible” a process is.

Through 1895, Planck’s research

- gave a very general formulation of the second law, and then
- applied it in the emerging discipline of physical chemistry. Planck used the second law as a tool for understanding chemical reactions

The reception was not all Planck could hope for:

> **“An important scientific innovation rarely makes its way by gradually winning over and converting its opponents: it rarely happens that Saul become Paul. What does happen is that its opponents gradually die out and that the growing generation is familiarized with the idea from the beginning: another instance of the fact that the future lies with youth.”**
>
> (Planck 1936, p. 97).

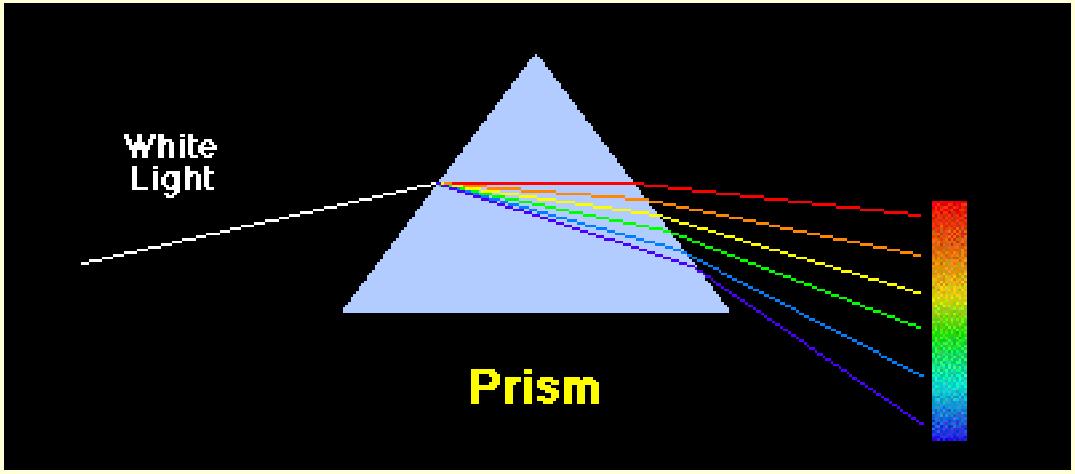
Black Body Radiation

In 1895, Planck turned to a new problem, and he hoped, a new application of the second law:

What is the character of the heat and light emitted from a hot object?

“Newton’s Rainbow”

White light from a hot object (the sun), passing through a prism, is separated into a continuous spectrum of colors.



Black Body Radiation

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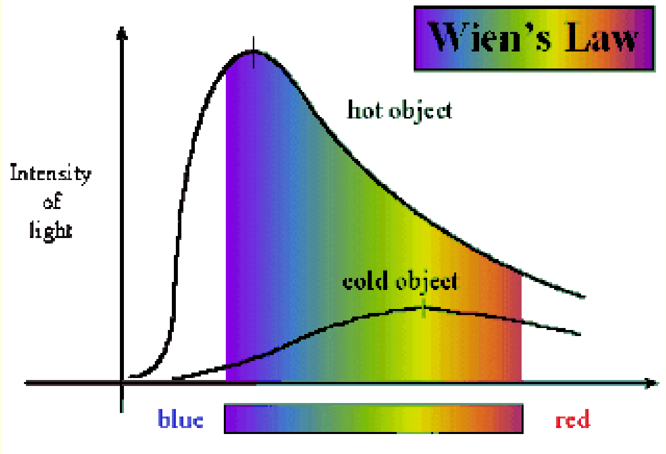
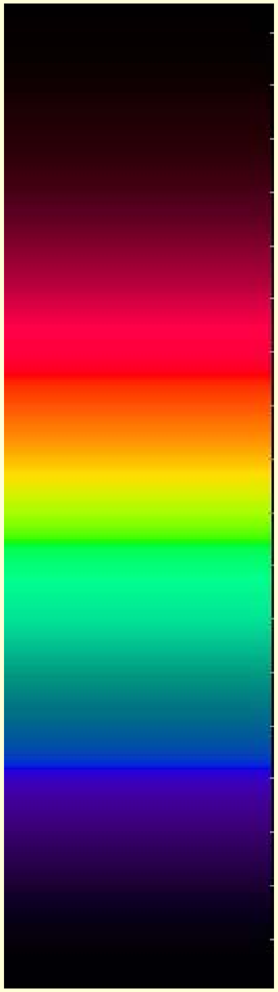


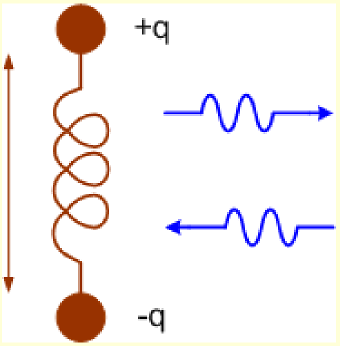
Wilhelm Wien

All colors in this spectrum are not equally intense--there is a peak brightness that depends on the temperature of the hot object.

- Examples: We infer the temperature of a red-hot iron bar, or a yellowish tungsten light bulb from the colors. One can even infer the temperature of stars from their colors!



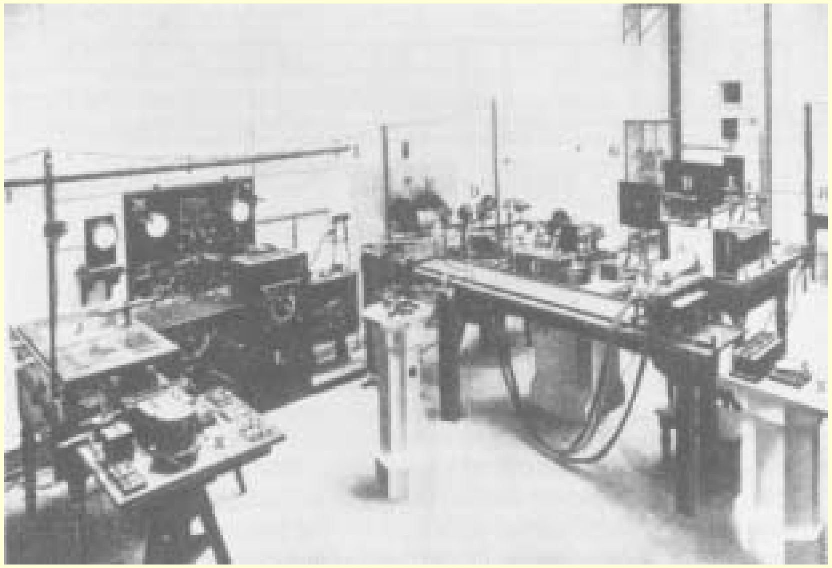
Planck's Problem

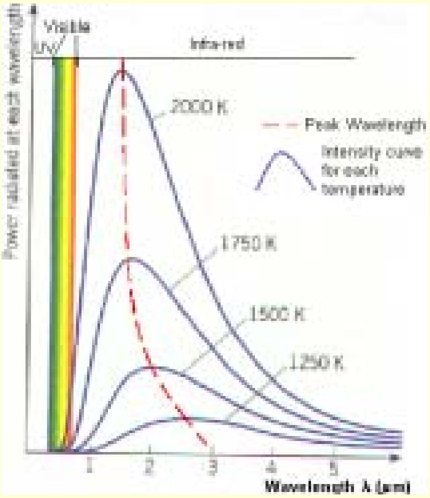


Consider a box filled with thermal radiation:

Show that the Second Law of Thermodynamics applied to a microscopic "resonator" (an oscillator representing the walls of the box) exchanging energy with electromagnetic radiation (light and heat), leads to an equation for the intensity (brightness) of the black body spectrum.

CONSTRAINT: The excellent and ever-improving data provided by experimentalists, mostly at the Physikalisch-Technische Reichsanstalt in Charlottenburg, close by Planck in Berlin.





Black Body Radiation: Planck's First Solution

Planck began this work in 1895, and by late 1899, had a result:

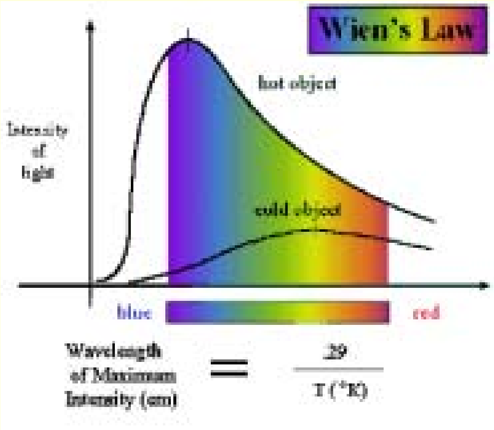
$$\text{Radiation Intensity (brightness)} = \frac{8\pi \boldsymbol{a}}{c^3}\nu^3 e^{-\boldsymbol{b}\nu / T}$$

This result is called Wien's Law because it had **already been derived, in 1894**, by Planck's colleague and good friend Wilhelm Wien. But Wien's derivation had been dubious; Planck thought his derivation, based on the Second Law, was far more rigorous. Of it, he said:

> **I believe that the definitions given for the entropy of radiation and also the Wien distribution law for radiation … are necessary consequences of the second law of thermodynamics.**

Planck called the constants ***a*** and ***b*** "universal constants", and said that they

> **necessarily retain their significance for all times and places, even alien and non-human ones.**




Wilhelm Wien

The Experimentalists improve the measurements

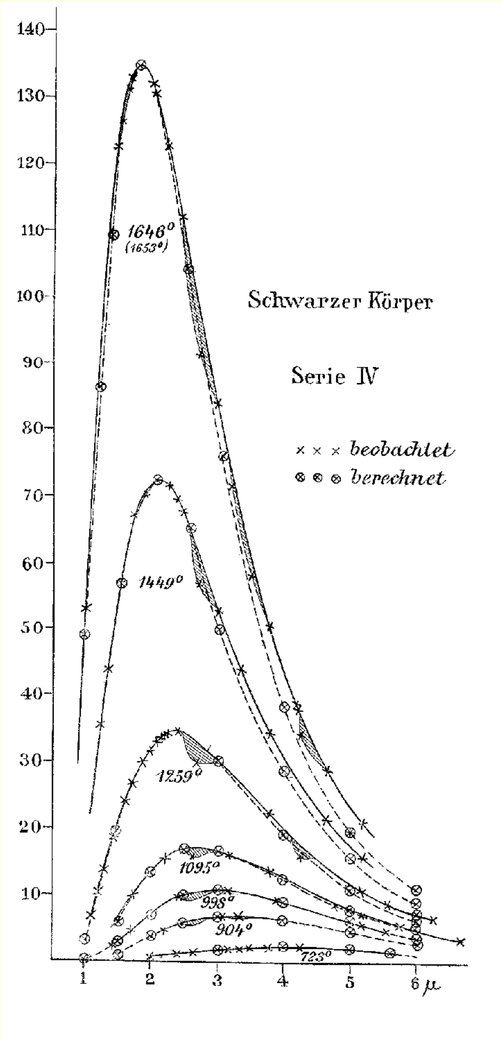
At about the same time, two experimentalists working at the PTR were finding that Wien's law might not be correct.

The discrepancies between the observed and … calculated energy curves are not due to haphazard experimental errors.

Herr Planck has declared that this law [Wien's Law] is a necessary consequence … of the second law of thermodynamics. As far as we can see, Planck's theory would be more compelling if it could be shown that every departure from the above equation leads to an expression for the entropy that contradicts the entropy law.

Before we pass judgment on the applicability of the Wien-Planck equation, we think it necessary to extend the measurements to larger temperature intervals and wavelengths.

Otto Lummer and Ernst Pringsheim
November 1899



Planck improves the derivation!

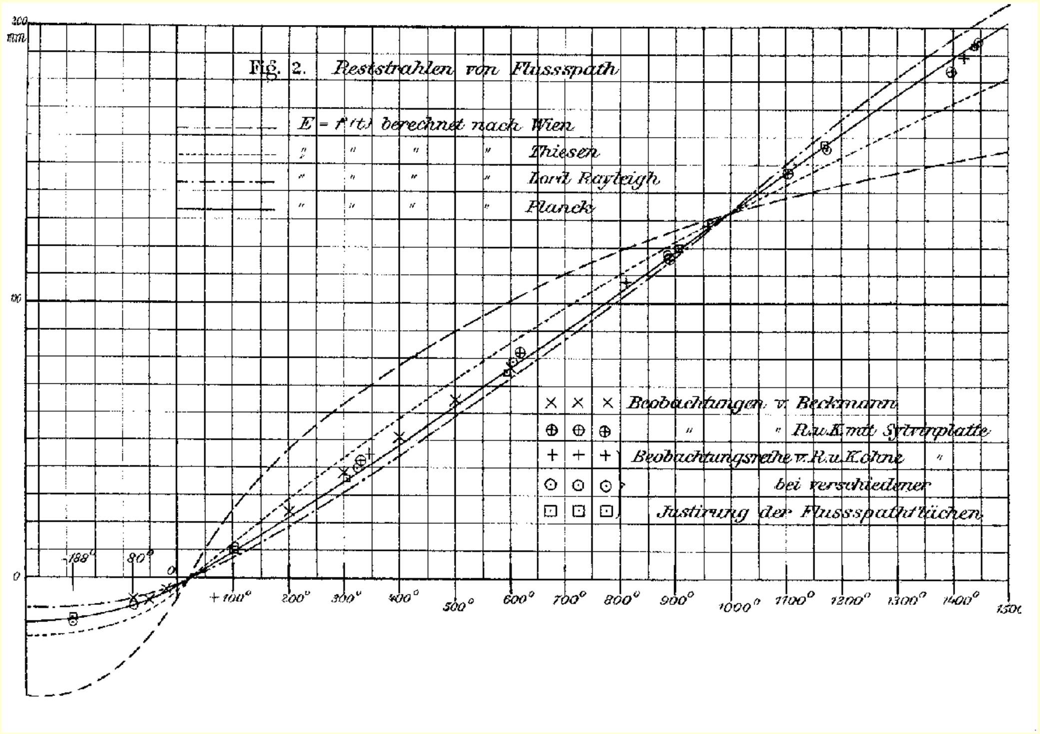
Planck's response in March 1900: He explained even more carefully the foundations of his theory and why he thought it was on solid ground.

He went on to **improve** the argument, giving a derivation of a central result that in 1899 he had introduced only as a definition. He concluded:

> **My view of the significance of [this result (Wien's Law)] has become even stronger, even if the foundation on which it rests has in part shifted somewhat.**

Even better measurements

By the fall of 1900, new measurements by Heinrich Rubens and Ferdinand Kurlbaum made the discrepancies between the data and Wien's Law inescapable.



Planck observed that the failure of his improved March derivation was "not easily understandable."

Planck's new law

In October 1900 at a meeting of the German Physical Society, Planck announced a new equation that was fully consistent with all of the PTR observations:

$$\text{Radiation Intensity (brightness)} = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

$$h = 6.63 \times 10^{-34}$$

$$k = 1.38 \times 10^{-23}$$

> **The very next morning I received a visit from my colleague Rubens. He came to tell me that after the conclusion of the meeting he had that very night checked my formula against the results of his measurements, and found a satisfactory concordance at every point. … Later measurements, too, confirmed my radiation formula again and again.**
>
> **But even if the absolutely precise validity of the radiation fomula is taken for granted, so long as it had merely the standing of a law disclosed by a lucky intuition, it could not be expected to possess more than a formal significance. For this reason, on the very day when I formulated this law, I began to devote myself to the task of investing it with a true physical meaning.**

Planck, *Scientific Autobiography*

To do so, Planck turned to the work of the Viennese physicist Ludwig Boltzmann

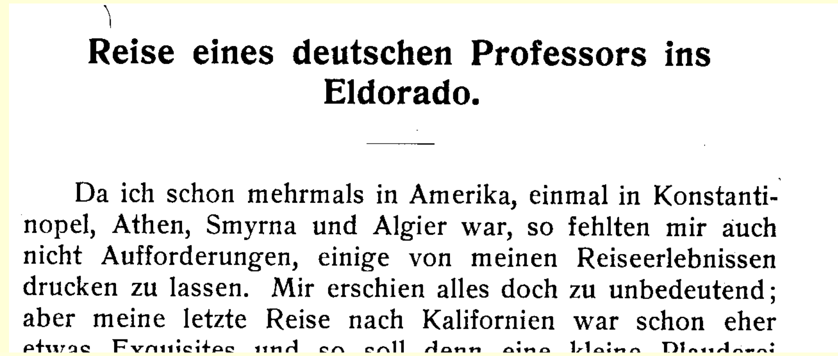
Ludwig Boltzmann
1844 – 1906

Much of Boltzmann's career had likewise been devoted to the second law of thermodynamics, but from a different perspective.

Where Planck was for a long time dubious about the existence of atoms, Boltzmann took them as central.

Boltzmann in California

In 1905, Boltzmann was invited to lecture at the summer school of the University of California at Berkeley



Reise eines deutschen Professors ins Eldorado.

Da ich schon mehrmals in Amerika, einmal in Konstantinopel, Athen, Smyrna und Algier war, so fehlten mir auch nicht Aufforderungen, einige von meinen Reiseerlebnissen drucken zu lassen. Mir erschien alles doch zu unbedeutend; aber meine letzte Reise nach Kalifornien war schon eher etwas Exquisites und so soll denn eine kleine Plauderei

Yes, America will achieve great things still. I believe in this nation, even though I saw it engaged in an occupation at which it does not excel: … a theoretical physics seminar.

My stomach troubles returned at this point. It is possible to get wine in the restaurant car, but grudgingly … It takes an eternity, and … one has to sit in front of the iced water with a parched throat. I succumbed to temptation (angels would have fallen) and drank some of the poison.

Journey of a German Professor

This is the form my English Conversation took:

I: When will lunch be served?

He: ieeöö

I: I beg you, could you say me, at what hour lunch will be served?

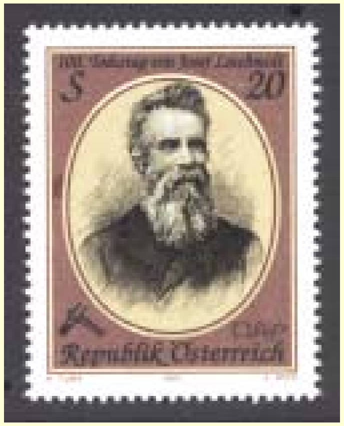
His splutterings began a good fifth deeper: aoouu

I realize the error in my plan of attach and cry despairingly: lönch, lanch, lonch, launch and so on. I produce vowels that would never be found in Gutenberg's type-cast. Now his face shows a glimmer of understanding: *ah, loanch?*

… And now I was supposed to give 30 lectures in this language!

Subsequently I was invited somewhere or other every Saturday or Sunday. The first invitation was to Mrs. Hearst's splendid estate near Livermore. Who is Mrs. Hearst? It is not easy to explain to a European. The nearest to the truth is that she is the University of Berkeley. In Europe the Alma Mater is a classical idealized figure. In America, she is a real figure, and what is most important, has real millions of dollars … my American visit was paid for with her money, naturally.

In remembrance of Josef Loschmidt



In 1895, Boltzmann delivered an obituary for his good friend and colleague, the Viennese physicist and chemist Josef Loschmidt, who we still remember for his calculation of the size of a molecule:

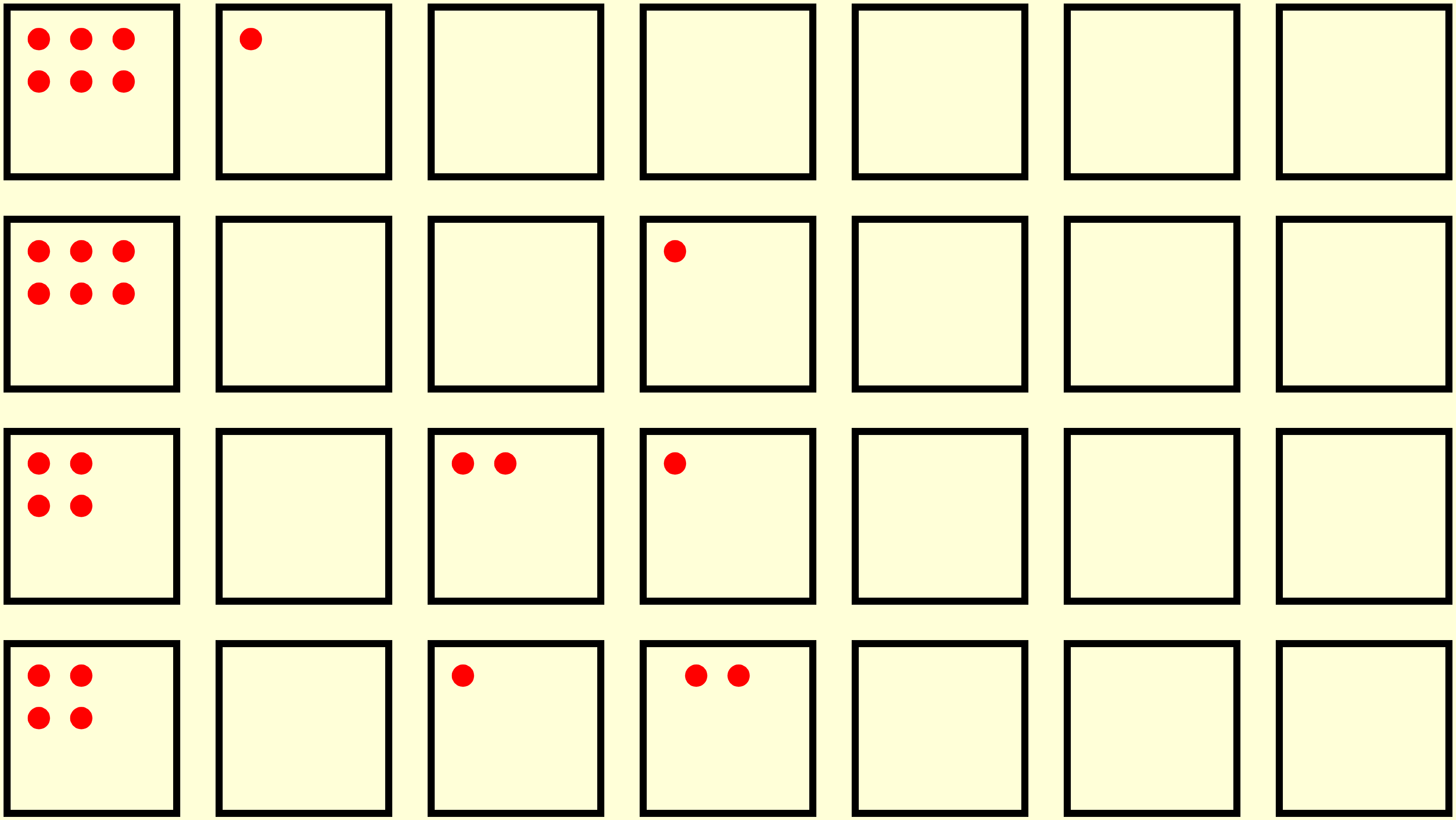
> **Now Loschmidt's body is scattered into its atoms. How many, we can calculate from the principles he established, and so that it is not absent in a talk in honor of an experimental physicist, I have written the number on the blackboard (10 quadrillion = 10^{25}).**

In 1876, Loschmidt had challenged Boltzmann's establishment of the second law on a molecular basis. How could it be that the laws governing the motion of molecules, which do not show a direction in time, lead to the second law of thermodynamics, which does?

Boltzmann responded with a paper in 1877 that showed how the laws of probability lead to just such a conclusion. This paper was not widely known in 1900, but Planck had come across it, and used it in late 1900.

Boltzmann's 1877 calculation

Suppose we divide the energy of a gas into **finite** units, and ask how many ways we could partition those energy units among the molecules of a gas. Suppose we have 7 molecules, and 7 energy units. Here are a few ways of doing the partition:



Boltzmann's 1877 calculation

Boltzmann discovered that for large numbers of molecules, the overwhelming majority of these partitions corresponded to the "equilibrium state"— that is, a gas sitting peacefully in a container and behaving in the expected way. Other possibilities were not impossible, but enormously improbable. In this way of thinking (one **not** congenial to Planck!), the second law was not absolute, but only overwhelmingly probable. One physics text summarized this situation with the following quotation:

"What never?" "No never!"

"What never?" "Well, hardly ever!"

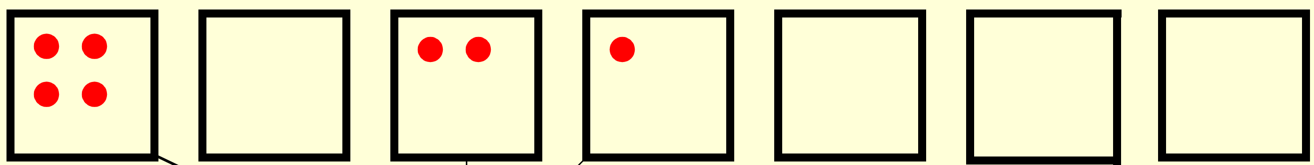
H.M.S. Pinafore, as quoted in Mayer and Mayer, *Statistical Mechanics*

But there was more to be done. Boltzmann knew that his calculation was **artificial**— energy, after all, isn't divided into finite units. So having made his point, he went on to make his calculation more realistic.

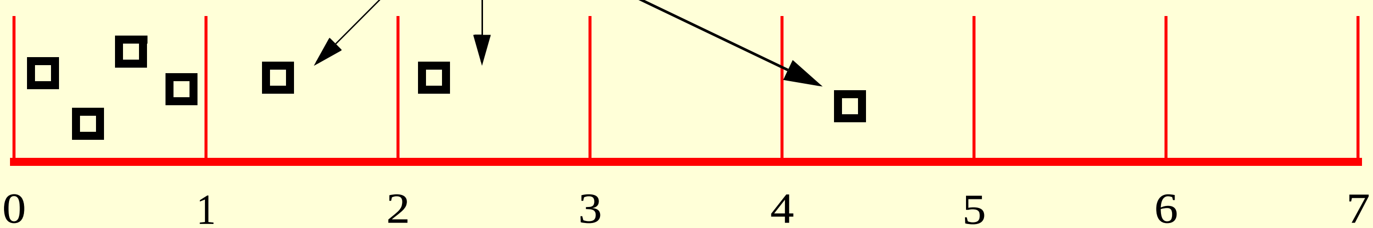
Boltzmann's more realistic calculation

Instead of distributing energy units among molecules, Boltzmann treated energy as continuous, but divided into "cells"--thus, if a molecule had an energy between 0 and 1 energy unit, it was in the first cell; if between 1 and 2 energy units, the second cell; and so on.

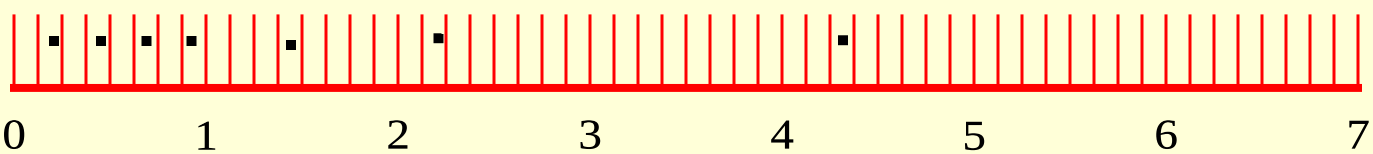
Thus, instead of this picture



Boltzmann imagined this one: Instead of energy units, molecules partitioned among "cells." Below, there are 4 molecules with energies between 0 and 1 energy unit, none between 1 and 2 energy units, and so on. The mathematics is the same.

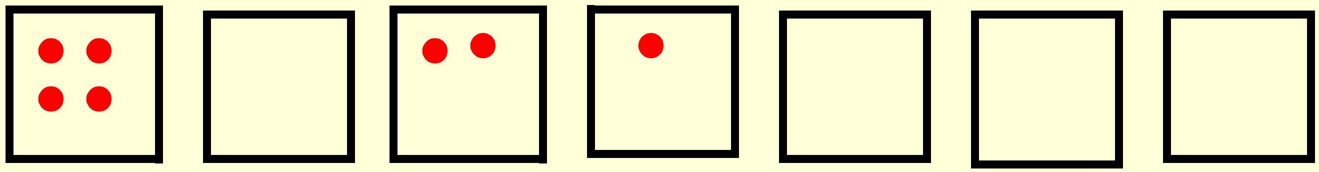


And then as a final step, he let the cells shrink to vanishingly small

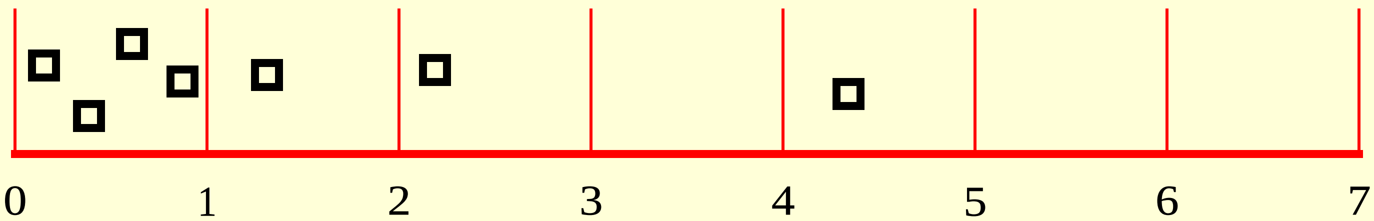


Note again the three steps

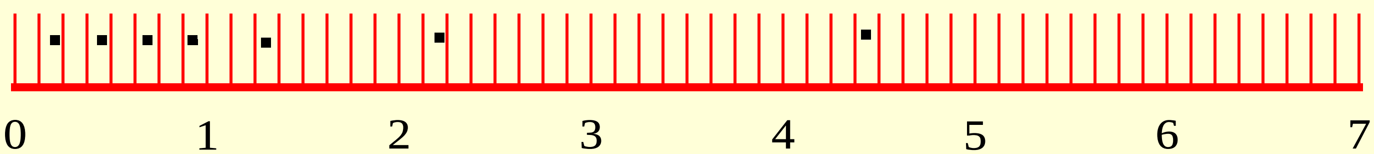
Step 1: Distribute energy units among molecules



Step 2: More realistically, distribute molecules among energy "cells"--so **energy is continuous again**!



Step 3: Let the cells shrink to vanishingly small



Result: Boltzmann recovered all of the standard properties of an ideal gas, and so took his probabilistic assumptions as justified!

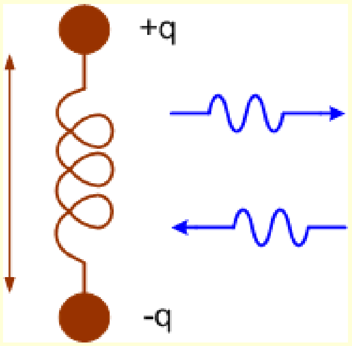
Boltzmann had shown that the Second Law (and in particular, the **entropy**) was related to probability.

Ironically, the equation that expresses that relationship

$$S = k \log W$$

was first written down by Max Planck in late 1900.

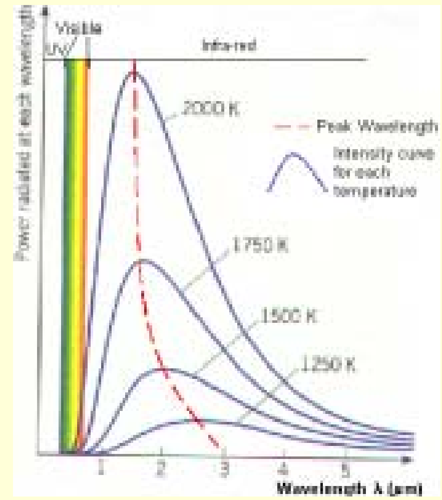
Planck adopted Boltzmann's method



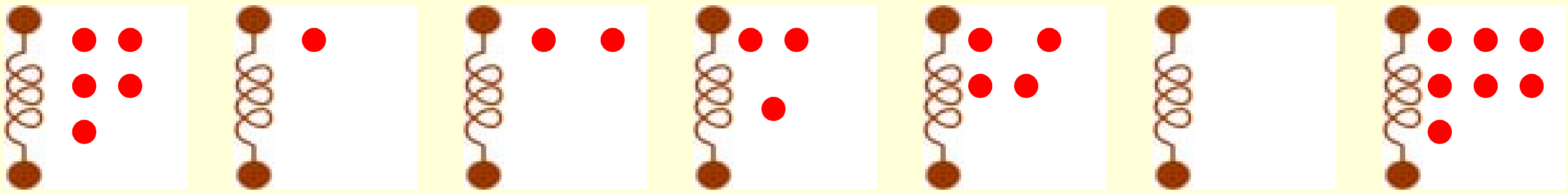
resonator

Recall that Planck was imagining "resonators"--microscopic oscillators--exchanging energy with heat radiation.

He now imagined the energy of this system divided into finite **energy elements,** and calculated how many ways a large number of energy elements could be partitioned among a collection of resonators.



Example: One way to distribute 22 energy elements among 7 resonators



He was thus using Boltzmann's relation between entropy and probability to analyze his problem--derive his new expression for the shape of the black body energy distribution curve.

It worked!

Using Boltzmann's methods, Planck was able to derive his new equation for the brightness of black body radiation. He reported the new derivation in three papers published in late 1900 and early 1901.

$$\text{Radiation Intensity (brightness)} = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1}$$

$$h = 6.63 \times 10^{-34}$$

$$k = 1.38 \times 10^{-23}$$

But

Planck never went beyond the **first step** of Boltzmann's derivation.

In order for Planck's derivation to work, the energy elements **had to** remain **finite**, and, it turned out, their **size** was determined by his new constant h. In short: Planck had distributed finite energy elements of fixed size over the resonators, so that each resonator had an integral number of energy elements.

Had Planck introduced the quantum into physics?

What did Planck say about this state of affairs in 1900?

What did Planck say about his derivation in 1900?

What did Planck say about his derivation in 1900?

$$h = 6.63 \times 10^{-34}$$

$$k = 1.38 \times 10^{-23}$$

What did Planck think he was doing in 1900?

The Textbooks

Planck … had to make a drastic, quite unjustified assumption: that the oscillators could only emit and absorb energy of frequency f in units of hf, where h is a new universal constant .... Planck called these energy units **quanta.**

Planck departed radically from classical physics. He postulated that the energies of the oscillators are quantized… the only permitted values of the energy are… $0, h\nu, 2h\nu, 3h\nu$. All other values of the energy are forbidden. The constant h is Planck's constant.

The Historians

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Reading his papers today, we are struck by how little emphasis he gave to this radical idea. …

Martin J. Klein

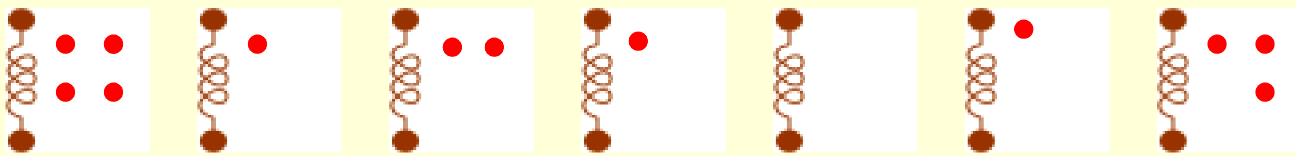
My point is not that Planck doubted the reality of quantization or that he regarded it as a formality to be eliminated during the further development of his theory. Rather, I am claiming that the concept of restricted resonator energy played no role in his thought.

Planck's resonators ... absorbed and emitted radiation continuously.... His theory was still classical.

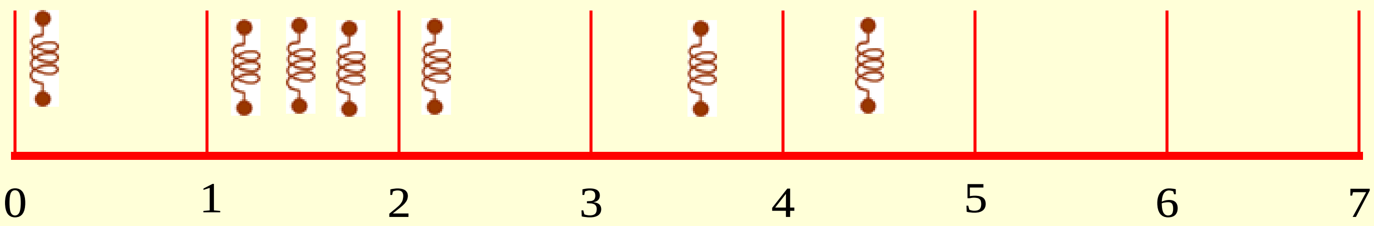
Thomas S. Kuhn

Kuhn's central argument:

Planck said this (Boltzmann, Step 1):



But he meant this (Boltzmann, Step 2)



The mathematics is the same in both cases; but in the second, there is no suggestion of quanta or discontinuous energies.

Moreover, Boltzmann's 1877 approach was unfamiliar to most physicists in 1900, and Planck's papers drew little reaction for several years.

Planck's 1906 *Lectures*

Among other evidence, Kuhn points to Planck's 1906 *Lectures on Thermal Radiation*, where Planck makes two allusions to continuous energies.

[Planck speaks of] **… the number of resonators that hold a definite quantity of energy (better: that lie in a definite "energy cell.")**

Let us consider … the probability that the energy of a resonator lies between the values U and $U + \Delta U$ …

Otherwise, the derivation is exactly the same as in 1900: energy elements distributed among resonators.

More from Planck's 1906 *Lectures*

from 1906:

[Planck speaks of] **… the number of resonators that hold a definite quantity of energy (better: that lie in a definite "energy cell."**

Let us consider … the probability that the energy of a resonator lies between the values *U* and *U* + Δ*U* …

Otherwise, the derivation is exactly the same as in 1900.

Whatever else Planck may have thought, he knew that his new constant *h* was unexplained and would lead to new physics.

There can be no doubt that the constant *h* plays a definite role at an emission center of the elementary oscillation process …. [The] thermodynamics of radiation will have arrived at an entirely satisfactory conclusion only when the constant *h* is understood in its full universal significance.

Naturally, the action element *h* must receive a direct electrodynamic meaning; but of what sort remains for the present an open question.

…one might possibly consider, in the place of the simple linear differential equation [considered here], another oscillation law that is even better adapted to the processes in nature.

Planck's Correspondence

Letter from Planck to Wien, March 1907

… in general quite a few molecules do not emit at all while others emit either one full energy quantum or several energy quanta.

There are other examples from this period. Planck several times refers to energy quanta, but never to continuous energies.

Planck in 1908

By 1908, everyone agrees that Planck was taking quanta seriously.

from 1906:

[Planck speaks of] **… the number of resonators that hold a definite quantity of energy (better: that lie in a definite "energy cell.")**

Let us consider … the probability that the energy of a resonator lies between the values *U* and *U* + Δ*U* …

Otherwise, the derivation is exactly the same as in 1900.

from a 1908 letter from Planck to H. A. Lorentz:

In sum, I might therefore say, I make two assumptions.

1. The energy of a resonator is $gh\nu$, (*g* a whole number or zero.)

Planck in 1909

In 1909, Planck gave a series of lectures at Columbia University in New York

from 1906:

[Planck speaks of] **… the number of resonators that hold a definite quantity of energy (better: that lie in a definite “energy domain.”)** [or cell]

Let us consider … the probability that the energy of a resonator lies between the values *U* and *U* + Δ*U* …

Otherwise, the derivation is exactly the same as in 1900.

from a 1908 letter from Planck to H. A. Lorentz:

In sum, I might therefore say, I make two assumptions.

1. The energy of a resonator is *gh*ν, (*g* a whole number or zero.)

from the 1909 Columbia Lectures:

… we desire to find the … probability that the energy of a resonator shall lie between *U* and *U* + Δ*U* …

Did Einstein get it right?

In 1906, an obscure Swiss Patent Office clerk wrote a review of Planck's book for the *Annalen der Physik*

The author repeatedly points to the necessity of introducing this universal constant *h* and emphasizes the importance of a physical interpretation (not given in the book) of the latter.